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Analysis of Students' Algebraic Reasoning Processes in Expanding Mathematical Expressions in Nonroutine Problems

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Abstract: This study investigates the algebraic reasoning processes of university students when expanding mathematical expressions in the context of nonroutine problem-solving. The research adopts a qualitative approach to explore how students interpret algebraic structures, apply symbolic transformations, and construct logical explanations while working through unfamiliar tasks. Data were collected through written tests, task-based interviews, and detailed analysis of students' solution strategies. The findings reveal significant variation in students' ability to generalize patterns, recognize structural relationships, and justify algebraic procedures. Students with strong conceptual understanding demonstrated flexible reasoning, coherent explanations, and appropriate use of algebraic properties. In contrast, students who relied heavily on procedural rules often struggled with symbolic manipulation, produced fragmented reasoning, and exhibited misconceptions related to variables and distributive operations. These results highlight the importance of fostering conceptual understanding, metacognitive awareness, and reasoning-oriented instruction in university mathematics. The study provides insights for educators seeking to design learning environments that promote deeper algebraic thinking and enhance students' ability to solve complex, nonroutine problems.

Keywords: Algebraic Reasoning; Mathematical Expression Expansion; Nonroutine Problem-Solving; Qualitative Analysis; Symbolic Manipulation

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1. Introduction

The development of algebraic reasoning has become a central focus in contemporary mathematics education, particularly as higher education institutions aim to cultivate students' abilities to engage in advanced problem-solving and analytical thinking. Algebraic reasoning is not merely the manipulation of symbols; it represents a cognitive process that allows individuals to interpret, generalize, and transform mathematical relationships. In university settings, this capacity becomes increasingly important, as students must work with complex mathematical ideas and apply them in various contexts. Consequently, understanding how students develop and use algebraic reasoning is essential for assessing the effectiveness of instructional approaches and identifying areas that require pedagogical improvement [1].

Nonroutine mathematical problems provide a meaningful platform for evaluating algebraic reasoning, as they require students to move beyond memorized procedures and employ deeper conceptual understanding. Unlike routine tasks that follow predictable patterns, nonroutine problems challenge students to analyze unfamiliar structures, construct strategies, and justify their reasoning. These tasks therefore serve as an effective lens for examining how students interpret expressions, make transformations, and engage in multi-step reasoning processes. By studying students' performance on nonroutine problems, educators and researchers can gain insight into the sophistication and flexibility of their algebraic thinking [2].

Although many university students have been exposed to algebra for years, research indicates that their ability to manipulate and interpret algebraic expressions meaningfully remains limited. Students often rely heavily on procedural approaches without fully understanding the underlying concepts. When faced with nonroutine problems, these surface-level strategies frequently fail, revealing gaps in conceptual comprehension and reasoning fluency. Such challenges suggest that traditional instructional methods may not sufficiently support the development of algebraic reasoning, particularly in areas requiring abstraction and symbolic manipulation.

In the context of higher education, the ability to expand or elaborate on algebraic expressions is critical for success in various mathematical domains, including calculus, linear algebra, and discrete mathematics [3]. Expression manipulation forms the basis for deriving formulas, solving equations, analyzing functions, and interpreting mathematical models. Yet, students' errors in expansion and transformation often stem from deeper cognitive difficulties, such as misunderstanding symbolic representations or failing to connect algebraic procedures with real mathematical meaning. These difficulties become even more pronounced when problems are embedded in unfamiliar or complex contexts [4].

The cognitive processes underlying algebraic reasoning involve several interconnected skills, such as recognizing patterns, constructing relationships, performing symbolic transformations, and validating solutions. When students engage with nonroutine tasks, these skills must work in concert, requiring them to think flexibly and adaptively. However, studies suggest that many students struggle to coordinate these cognitive components effectively. By exploring how they approach such problems, researchers can better understand the nature of these difficulties and the reasoning patterns that emerge during problem solving.

A qualitative approach offers valuable insights into the nuances of students' algebraic reasoning, as it allows for the exploration of thought processes rather than simply evaluating correctness. Through interviews, think-aloud protocols, and analysis of written work, researchers can uncover students' strategies, misconceptions, and underlying conceptual frameworks. This approach enables a deeper understanding of how students interpret symbolic expressions, navigate problem-solving pathways, and justify their choices when working through nonroutine mathematical challenges [5].

Existing literature highlights several common issues in students' algebraic reasoning, such as overgeneralization of rules, misinterpretation of symbolic notation, and reliance on rote procedures. However, these findings are often derived from general assessments or routine tasks, leaving a gap in understanding how students reason in more demanding problem-solving situations. There remains a need to investigate how students break down mathematical expressions, decide on appropriate transformations, and evaluate the coherence of their solutions under nonroutine conditions [6].

Furthermore, the increasing emphasis on higher-order thinking skills within university mathematics curricula underscores the importance of cultivating strong algebraic reasoning abilities. As the complexity of mathematical content rises, students must be able to engage in meaningful manipulation and interpretation of algebraic expressions. Understanding how they navigate these demands in the context of unfamiliar or complex problems can inform the design of instructional interventions that better support their learning needs.

Given these considerations, examining university students' algebraic reasoning in the context of nonroutine problem solving is both timely and necessary. Such studies can contribute to the growing body of knowledge on mathematical cognition, provide insights into students' conceptual understanding, and highlight the cognitive challenges that arise when students attempt to elaborate or expand algebraic expressions in novel situations. These insights can, in turn, guide the development of teaching strategies that foster deeper, more flexible reasoning abilities.

This study aims to analyze the processes through which university students engage in algebraic reasoning when elaborating mathematical expressions in nonroutine problems. By exploring their thought patterns, strategies, and conceptual challenges, the research seeks to provide a comprehensive understanding of the cognitive mechanisms underlying successful and unsuccessful reasoning. Ultimately, the findings are expected to support the enhancement of instructional practices that promote meaningful algebraic thinking at the university level.

2. Literature review

Algebraic Reasoning in Higher Education

Algebraic reasoning has increasingly been acknowledged as a cornerstone of mathematical competence, especially in higher education where students encounter complex abstractions and formal symbolic systems. This form of reasoning enables learners to move beyond arithmetic thinking and engage with generalized mathematical ideas, patterns, and structures. As university curricula place greater emphasis on abstraction, students are expected to reason algebraically in ways that support deeper comprehension and advanced problem solving. The ability to think relationally, recognize structures, and manipulate algebraic forms becomes essential for navigating the demands of higher-level mathematics [7].

Researchers commonly describe algebraic reasoning as a multifaceted cognitive process that integrates conceptual understanding and procedural fluency. Conceptual knowledge allows students to grasp the underlying principles of algebraic operations, while procedural fluency supports the efficient execution of symbolic tasks. However, developing a balance between these two dimensions remains a persistent challenge. Students who rely primarily on procedural skills often struggle to explain or justify their work, demonstrating gaps in understanding that can impede their progress in university mathematics courses. Therefore, fostering both aspects is critical for cultivating robust algebraic reasoning.

A key characteristic of algebraic reasoning is the ability to generalize mathematical relationships. Generalization allows students to identify patterns and regularities, making abstractions that lead to broader principles applicable across mathematical contexts. This generalizing process is vital in higher education, where students are required to work with functions, sequences, and symbolic forms that represent infinite or variable quantities. Without strong generalization skills, students may resort to rote procedures that limit their capacity to engage meaningfully with advanced mathematical ideas [8].

Another essential component of algebraic reasoning is the interpretation of symbolic expressions and structures. Symbolic notation serves as a concise representation of mathematical ideas, but its compact form can obscure meaning for students who lack conceptual grounding. Misinterpretations often arise when students view symbols merely as objects to manipulate rather than as representations of relationships and processes. In higher education, where symbolic complexity increases, difficulties in interpretation can significantly hinder students' ability to solve problems and understand new material.

The ability to manipulate symbolic forms coherently is also central to algebraic reasoning. Symbolic manipulation includes transforming expressions, solving equations, and rewriting mathematical forms to reveal underlying relationships. While many students can perform manipulations by following memorized procedures, meaningful reasoning requires understanding how and why these transformations work. When students treat symbolic manipulation as mechanical steps, they often make errors that reflect deeper conceptual misunderstandings. Ensuring that symbolic manipulations are grounded in meaning is therefore essential for developing strong algebraic reasoning.

Justifying procedures and solutions represents another critical dimension of algebraic reasoning. In higher education, students are expected not only to perform correct operations but also to justify their reasoning logically. Justification involves linking symbolic transformations to mathematical principles, demonstrating awareness of structure, and articulating the rationale behind chosen strategies. Without explicit attention to justification, students may rely on rote methods that fail when they encounter unfamiliar or complex problems. Encouraging justification supports the development of mathematical argumentation, a skill central to advanced mathematical thinking.

Transitions between mathematical representations play a vital role in supporting algebraic reasoning. Students must be able to move fluidly among symbolic, graphical, numerical, and verbal forms to construct meaningful interpretations of mathematical ideas. Research shows that flexible representational thinking enhances students' ability to understand relationships and make informed decisions during problem solving. However, many students struggle with these transitions, particularly when faced with abstract representations that require interpretation beyond surface-level features. Strengthening representational fluency is thus an important component of developing algebraic reasoning.

Within the university context, algebraic reasoning serves as a foundation for understanding advanced topics such as calculus, linear algebra, abstract algebra, and mathematical modeling. These domains require students to manipulate complex symbolic structures, interpret functional relationships, and generalize concepts across contexts. Students who lack strong algebraic reasoning often face significant barriers in these subjects, leading to frustration and poor performance. Consequently, educators and researchers emphasize the importance of addressing foundational reasoning skills early in higher education to support long-term mathematical success.

Despite its importance, many studies highlight persistent challenges in students' development of algebraic reasoning. Students frequently exhibit proficiency in executing procedures while lacking conceptual understanding, a discrepancy that becomes especially visible in problem-solving situations requiring flexible reasoning. This imbalance underscores the need for instructional approaches that integrate procedural practice with opportunities for conceptual exploration, pattern recognition, and structural analysis. Addressing these challenges is essential for helping students develop deeper and more adaptable reasoning abilities.

Given the complexity and importance of algebraic reasoning, understanding the cognitive processes that support or hinder its development remains a significant focus in mathematics education research. By examining how students approach algebraic tasks, interpret symbols, and justify their reasoning, researchers can identify patterns that inform instructional improvement. Enhancing students' algebraic reasoning is not only necessary for their success in university mathematics but also fundamental for developing the analytical skills required in broader scientific and technological fields.

Expansion and Transformation of Mathematical Expressions

The expansion and transformation of algebraic expressions form a central component of mathematical activity, particularly in higher education where symbolic complexity increases significantly. These skills enable students to rewrite mathematical statements in forms that reveal underlying relationships, simplify problem structures, and support further analytical steps. Rather than serving as isolated procedures, expansion and transformation function as foundational elements of mathematical reasoning, allowing learners to navigate between equivalent representations and uncover new insights. As students progress into more advanced mathematics, their ability to manipulate expressions coherently becomes increasingly important for understanding and constructing mathematical arguments.

Effective expansion of algebraic expressions requires students to recognize structural features within the expressions they manipulate. This involves identifying patterns, such as binomial forms or distributive structures, and understanding how symbolic components relate to one another. When students rely solely on surface-level features, they often miss deeper structural cues that guide appropriate transformation. Research shows that students who possess strong structural awareness are better able to generalize rules, detect errors, and adapt their reasoning to unfamiliar problems. Structural insight thus plays a crucial role in enabling students to approach expansion tasks with flexibility and understanding [9].

Despite the importance of structural awareness, many students struggle with the expansion and transformation of expressions due to an overreliance on memorized procedures. Such procedural dependence often leads to common errors, including misapplication of distributive rules or confusion between coefficients and variables. These mistakes reflect deeper misconceptions about how algebraic symbols represent quantities and relationships. When students fail to grasp the conceptual foundations of expansion, symbolic manipulation becomes disjointed and inconsistent. Addressing these conceptual gaps is essential for strengthening students' reasoning and preventing the recurrence of systematic errors.

The ability to transform algebraic expressions meaningfully also involves anticipating the consequences of symbolic operations. Students must understand how each transformation affects the overall mathematical meaning of an expression, including equivalence, functional relationships, and domain restrictions. This type of anticipatory reasoning is particularly important when solving nonroutine problems, where inappropriate transformations can lead students away from viable solution paths. Research suggests that students who possess strong anticipatory skills exhibit greater coherence in their symbolic reasoning and demonstrate higher levels of strategic decision-making during problem solving.

In advanced mathematical domains such as calculus, linear algebra, and mathematical modeling, the expansion and transformation of expressions are indispensable for constructing and simplifying complex representations. Students must frequently manipulate symbolic forms to derive formulas, analyze functions, or build mathematical models that reflect real-world phenomena. Proficiency in these skills allows students to engage deeply with material, draw meaningful conclusions, and articulate mathematical ideas clearly. Consequently, mastery of expression manipulation is not merely a technical requirement but a key component of the conceptual and analytical reasoning demanded in higher education mathematics [10].

Nonroutine Mathematical Problem Solving

Nonroutine mathematical problem solving occupies a central place in the development of higher-order thinking skills, as it requires students to move beyond familiar algorithms and apply flexible reasoning to unfamiliar situations. Unlike routine tasks that follow predictable and well-rehearsed patterns, nonroutine problems present novel structures or ambiguous information that compel learners to analyze relationships deeply and make sense of the mathematical context. This type of problem solving challenges students to engage in exploration, conjecture, and justification, processes that are essential for meaningful mathematical understanding. In higher education, exposure to nonroutine problems helps students cultivate the ability to adapt their reasoning to complex and less structured tasks.

A defining characteristic of nonroutine problems is the necessity for strategic decision-making. Students must determine which concepts, representations, or transformation strategies are relevant to the situation. Because these problems do not offer clear procedural cues, learners must draw on their conceptual knowledge to construct a pathway toward a solution. Research shows that students often struggle at this stage, particularly when their understanding of underlying concepts is incomplete. The absence of explicit instructions forces them to rely on their ability to interpret expressions, identify structural relationships, and formulate logical steps, making nonroutine tasks a valuable tool for assessing the depth of mathematical reasoning [11].

Nonroutine problem solving also highlights students' ability to make connections across different areas of mathematics. These tasks typically require integrating knowledge from multiple domains, such as algebra, geometry, and functions, in ways that standard procedural exercises do not. When students face problems that demand coordination of several ideas, their conceptual coherence or lack thereof becomes more visible. Difficulties in transferring knowledge across contexts often reveal fragmented understanding or misconceptions that may remain hidden during routine practice. Thus, nonroutine tasks offer insight into how well students can synthesize and apply mathematical concepts flexibly.

The role of metacognition becomes particularly important when students work on nonroutine problems. Successful problem solvers tend to monitor their reasoning, evaluate the plausibility of intermediate steps, and adjust strategies when necessary. These self-regulatory behaviors allow students to navigate complex solution paths and recover from unproductive approaches. However, many students lack metacognitive strategies or fail to apply them consistently, leading to difficulties in managing the cognitive demands of nonroutine tasks. Strengthening metacognitive skills can therefore enhance learners' capacity for effective problem solving and support the development of deeper mathematical thinking.

In the context of algebraic reasoning, nonroutine problems provide a rich environment for examining how students elaborate, interpret, and transform mathematical expressions. Such tasks require them to justify each symbolic manipulation and understand how transformations contribute to the overall solution structure. When students attempt to solve nonroutine problems, their reasoning becomes more transparent, allowing researchers and educators to analyze the cognitive processes involved. As a result, investigating students' performance on nonroutine tasks helps identify the reasoning patterns, challenges, and misconceptions that influence their ability to work with algebraic expressions at a higher conceptual level.

Hypothesis

The hypotheses of this study are grounded in the theoretical understanding that students' algebraic reasoning abilities play a crucial role in how they engage with expansion, transformation, and interpretation of mathematical expressions within nonroutine problem contexts. These hypotheses reflect the expectation that deeper conceptual understanding and structural awareness will lead to more accurate, coherent, and flexible reasoning processes. They also align with previous findings indicating that students' success in nonroutine tasks depends not only on procedural proficiency but also on their ability to generalize, justify, and adapt strategies according to the demands of the problem [12].

It is hypothesized that students with stronger algebraic reasoning will demonstrate more accurate and meaningful approaches when expanding or transforming algebraic expressions. Such students are expected to rely on their conceptual understanding of algebraic structures rather than solely on memorized steps. Their reasoning processes will likely show evidence of generalization, structural interpretation, and logical justification that support correct symbolic manipulation in unfamiliar contexts.

It is further hypothesized that students who struggle with expansion and transformation of expressions do so due to underlying conceptual gaps or misconceptions. These challenges are expected to manifest in errors such as misapplying the distributive property, misinterpreting variables, or altering symbolic structures in illogical ways. When confronted with nonroutine problems, these students are more likely to demonstrate rigid reasoning, limited flexibility, and reliance on superficial procedures that do not align with the problem's demands.

Another hypothesis proposes that students who employ diverse reasoning strategies—such as recognizing structural patterns, decomposing expressions, or generalizing symbolic relationships—will perform better on nonroutine tasks than those who depend on fixed or procedural approaches. The nature of nonroutine problems requires adaptive thinking, and students who can shift between strategies or integrate multiple forms of reasoning are expected to achieve more complete and coherent solutions.

A final hypothesis is that the level of students' algebraic reasoning can be inferred from the types of strategies, explanations, and justifications they provide during the problem-solving process. Students who articulate clear reasoning, consider alternative solution paths, and justify their transformations logically are anticipated to show more advanced reasoning capabilities. Conversely, students whose solutions lack explanation, display inconsistencies, or reveal unexamined assumptions are expected to demonstrate lower levels of algebraic reasoning.

3. Proposed Method

Research Design

Participants in this study consist of undergraduate mathematics education students who have completed foundational algebra courses and are currently enrolled in higher-level mathematics subjects. The selection is purposive, ensuring that participants possess the prerequisite knowledge to engage meaningfully with algebraic reasoning tasks [13]. Variation in academic performance is considered to capture a wide range of reasoning strategies and potential conceptual challenges. All participants are informed about the study's objectives and procedures and voluntarily agree to take part.

Participants

Data are collected using three primary instruments: nonroutine algebraic problem tasks, semi-structured interviews, and written solution analyses. The problem tasks are designed to require expansion, transformation, and interpretation of algebraic expressions beyond routine procedures, prompting students to reveal their reasoning processes. Semi-structured interviews allow the researcher to probe participants' thinking, clarify decision-making steps, and uncover implicit conceptual understandings. Written solutions serve as artifacts that capture symbolic manipulation and reveal patterns of accuracy, error, and structural interpretation.3.3. Data Collection Instruments [14].

Data Collection Procedure

Participants complete the nonroutine algebraic tasks individually in a controlled setting. Their written responses are collected, and follow-up interviews are conducted to gain further insights into their reasoning processes. During interviews, students are asked to explain their solution strategies, justify transformations, and reflect on challenges they encountered. All interviews are audio-recorded with participants' consent and later transcribed verbatim. Field notes are also used to document observations and contextual factors during the problem-solving session.

Data Analysis

Data analysis follows a thematic coding process guided by established qualitative analysis techniques. Transcripts, written solutions, and observational notes are reviewed repeatedly to identify recurring themes related to students' algebraic reasoning, conceptual understanding, and problem-solving strategies. Coding categories are developed inductively while also informed by theoretical frameworks on algebraic reasoning and expression transformation. The analysis seeks to map the reasoning patterns exhibited by students, identify misconceptions, and interpret how these elements influence performance in nonroutine tasks.

Trustworthiness of the Study

To ensure credibility, triangulation is applied by cross-referencing data from written solutions, interview transcripts, and researcher observations. Member checking is conducted by allowing participants to review and confirm the accuracy of interview interpretations. Transferability is addressed by providing detailed descriptions of the research context, participant characteristics, and data collection procedures. Dependability and confirmability are maintained through the use of an audit trail that documents analytical decisions and methodological steps throughout the research process [10].

4. Results and Discussion

Emerging Patterns of Algebraic Reasoning

Emerging patterns of algebraic reasoning among university students reveal significant variation in how they navigate symbolic forms, interpret mathematical structures, and generate generalized conclusions. The analysis shows that students differ substantially in their ability to recognize underlying algebraic relationships, which in turn influences the accuracy and coherence of their solutions. Some students exhibit strong structural awareness, while others rely heavily on surface-level procedures. These differences are not merely variations in skill but reflect deeper cognitive distinctions in how mathematical meaning is constructed and applied.

A prominent pattern involves the ability of students to identify algebraic structures embedded within expressions. Students with stronger reasoning consistently recognize forms such as binomials, factorable expressions, or functional relationships before manipulating symbols. They tend to analyze an expression holistically, determining how its components relate, rather than immediately applying an algorithmic rule. This structural recognition allows them to approach expansion or transformation tasks with intentionality, selecting strategies aligned with the expression's form.

Conversely, students who struggle with algebraic reasoning often fail to discern structural features and instead approach expressions linearly, term by term. Their reasoning tends to follow procedural scripts, such as "apply distributive property" or "simplify the terms," without first interpreting the global structure. As a result, they frequently misapply rules, especially when expressions deviate slightly from familiar patterns. These difficulties highlight a disconnect between symbolic manipulation and conceptual interpretation.

Symbolic understanding also emerges as a clear differentiator among students. Those exhibiting high symbolic fluency demonstrate an ability to interpret variables as generalizable quantities rather than as placeholders for numbers. They view symbolic expressions as representations of broader relationships and can justify transformations based on mathematical principles. Their written solutions reflect deliberate choices, such as preserving equivalence or maintaining structural integrity, which show sensitivity to the meaning of symbols.

In contrast, students with weaker symbolic reasoning often treat variables as static or context-bound. Their errors frequently involve misinterpreting the role of variables, introducing inconsistencies, or altering structures unintentionally. Such patterns suggest that their symbolic manipulations are not guided by conceptual reasoning but by fragmented procedural memory. These students often express uncertainty during interviews when asked to justify transformations, revealing that the meaning behind steps is not fully internalized.

Generalization abilities further distinguish students' levels of algebraic reasoning. Strong reasoners consistently identify recurring patterns, such as recognizing distributive structures or anticipating the outcome of an expansion based on prior experience. They are capable of extending reasoning beyond the immediate problem, articulating general principles that apply across different tasks. These students often verbalize reasoning such as "any binomial multiplied by another binomial will produce four terms initially, which can then be combined depending on like terms."

Students with limited generalization capacity tend to view each expression as an isolated entity. They require explicit prompts or familiar numerical examples to extend reasoning to broader cases. During interviews, they often struggle to articulate how strategies used in one problem might apply to another. Their reasoning remains localized, which restricts their ability to adapt to nonroutine tasks that demand flexible thinking.

Variation in depth of reasoning also appears in how students justify their transformations. Strong reasoners provide clear, logical explanations supported by algebraic principles. They show awareness of the implications of each symbolic manipulation and can reflect on potential alternative strategies. Their justifications demonstrate metacognitive engagement, indicating that they monitor and evaluate their own reasoning processes during problem solving.

In comparison, students exhibiting weaker reasoning rarely provide justifications beyond procedural statements. Their explanations tend to describe what they did ("I multiplied these terms") rather than why they did it. This lack of justification reveals limited conceptual grounding and suggests that their reasoning is primarily operational. When errors occur, they often cannot identify the source because they lack a framework for evaluating correctness beyond surface-level procedures.

These divergent patterns collectively illustrate the broad spectrum of algebraic reasoning present among students. The findings emphasize that algebraic proficiency is not merely the ability to manipulate symbols but the capacity to understand, interpret, and generalize mathematical structures. Recognizing these variations is crucial for designing instructional approaches that promote deeper reasoning and move students beyond procedural dependence.

To summarize emerging patterns, the following table highlights the contrast between strong and weak algebraic reasoning observed in the study:

Table 1. Comparison of Strong and Weak Algebraic Reasoning Patterns

Reasoning Dimension	Strong Algebraic Reasoning	Weak Algebraic Reasoning
Structural Awareness	Recognizes global structure before manipulation	Focuses on term-by-term procedures
Symbolic Interpretation	Treats variables as generalized quantities	Treats variables as static or numeric
Generalization	Identifies patterns, extends reasoning	Views each problem in isolation
Justification	Explains transformations conceptually	Describes steps without rationale
Flexibility	Adapts strategies to expression structure	Relies on fixed procedures

Conceptual Understanding versus Procedural Dependence

Conceptual understanding and procedural dependence represent two contrasting orientations that shape how students engage with algebraic expressions, particularly in tasks requiring expansion and transformation. The findings reveal that students vary widely in the balance they maintain between these orientations, with some demonstrating robust conceptual grounding while others rely heavily on memorized procedures. This distinction significantly influences the quality, accuracy, and flexibility of their mathematical reasoning.

Students who exhibit strong conceptual understanding approach algebraic expressions by first interpreting their structure and meaning. They demonstrate awareness of the relationships among terms, the implications of operations, and the purpose behind each step of manipulation. When expanding or transforming expressions, these students articulate why a particular rule applies and how it preserves mathematical equivalence. Their responses display coherence, with transformations connected logically to the underlying concepts guiding them. Such students also show the ability to predict the outcome of symbolic manipulations, reflecting a deeper internalization of algebraic principles.

In contrast, students dependent on procedural knowledge often approach tasks as sequences of steps to be executed rather than concepts to be understood. Their reasoning relies on recalling formulas or rules without fully grasping the conditions under which those rules apply. When encountering familiar expressions, they can produce correct answers; however, deviations from routines — such as nonstandard forms or embedded structures — quickly lead to confusion. Their written work often reveals mechanical application of distributive or simplification rules, sometimes resulting in transformations that break structural relationships or distort equivalence.

The limitations of procedural dependence become evident in the types of errors students frequently commit. A common error involves misapplication of the distributive property, such as distributing incorrectly across terms or failing to multiply every component. Students also demonstrate difficulties with variable interpretation, often treating variables inconsistently or applying numerical intuition where symbolic reasoning is required. These errors highlight a core issue: procedures performed without understanding can generate correct results only in narrow, predictable contexts but fail under more complex or unfamiliar conditions.

Conversely, students with stronger conceptual foundations exhibit fewer structural errors and demonstrate greater ability to self-correct. Their mistakes are often minor slips rather than misconceptions, and they can articulate the source of their errors during reflection. These students recognize when symbolic coherence is disrupted and adjust their reasoning accordingly. Their work reflects a dynamic interplay between conceptual insight and procedural execution, where procedures serve as tools rather than the basis of reasoning itself.

Another notable distinction is the students' flexibility in adapting their strategies. Conceptually oriented students shift between symbolic, structural, and verbal representations as needed. They can reframe expressions to reveal hidden patterns or simplify complex forms by drawing on conceptual connections. Those reliant on procedures tend to persist with a single strategy even when it becomes ineffective, indicating limited adaptability. Interviews show that procedural students often express frustration when a familiar algorithm does not yield progress, further illustrating their dependence on routine.

The contrast between conceptual and procedural approaches also appears in students' justifications. Conceptually grounded students provide explanations that reference properties, relationships, and the logic behind each step. Their reasoning is often relational, connecting one operation to another through an understanding of its purpose. Procedural explanations, however, center on describing actions taken rather than reasoning behind them. Statements such as "because that is the rule" or "I always multiply like this" reflect a lack of conceptual grounding and limited capacity for mathematical justification.

These patterns collectively reveal that procedural knowledge alone is insufficient for dealing with nonroutine algebraic tasks. Without conceptual understanding, students struggle to navigate expressions that require interpretation beyond surface-level manipulation. On the other hand, conceptual understanding enhances procedural fluency by providing a framework within which procedures gain meaning and direction. The findings therefore underscore the necessity of instructional approaches that integrate both forms of knowledge, while prioritizing conceptual depth to support transfer and adaptability.

To illustrate these contrasting orientations clearly, the following table summarizes the key differences observed between students dominated by conceptual understanding and those heavily dependent on procedures:

Table 2. Differences Between Conceptual Understanding and Procedural Dependence in Algebraic Reasoning

Dimension	Conceptual Understanding	Procedural Dependence
Approach to Expressions	Interprets structure before manipulation	Applies rules immediately without interpretation
Use of Procedures	Guided by understanding of concepts	Based on memorized steps or formulas
Error Patterns	Fewer structural errors; self-corrects	Misapplied rules; inconsistent variable use
Flexibility	Adapts strategies to expression structure	Relies on rigid, familiar procedures
Justification	Explains why each step is valid	Describes only what steps were taken

Strategies Used in Expanding and Transforming Expressions
The Multiple

Students' strategies for expanding and transforming algebraic expressions reveal a wide spectrum of reasoning approaches, ranging from highly structured manipulation to ad-hoc procedures that rely on isolated rules. As the data demonstrate, many students approach algebraic transformation with the intention of applying familiar techniques, yet their level of strategic control over these techniques varies considerably. Understanding these strategies provides critical insight into how students navigate symbolic complexity and how deeply they grasp the underlying algebraic structures.

One common strategy observed is rule-based expansion, where students apply distributive, associative, and commutative properties directly and systematically. This approach is often effective when students possess automaticity in procedural manipulation and can recall algebraic identities with ease. However, its success depends on whether students understand when specific rules are appropriate and how they connect to the structural features of the expression being manipulated. Those who rely solely on memorized steps tend to apply these rules rigidly, even when the expression requires a different or more flexible approach.

In contrast, some students demonstrate structure-driven strategies, focusing on identifying patterns within expressions before performing any manipulation. These students analyze the form of the expression—such as recognizing a binomial pattern, factoring schema, or opportunities to apply exponent rules—before committing to symbolic transformation. This strategy tends to result in more efficient solutions, especially in tasks involving nested structures or nonroutine manipulations. It also reflects a deeper conceptual understanding, as students purposefully seek structures that support meaningful transformations.

Another observed strategy involves recursive decomposition, where students break down complex expressions into smaller, more manageable subexpressions. Through stepwise simplification, they reconstruct the expression using known identities or previously solved components. This strategy is beneficial for handling multi-layered algebraic statements, such as those involving composition of functions or expressions with multiple variables. Students using this approach tend to maintain consistency in symbolic representation, reducing errors that arise from treating the expression as a single, unwieldy unit.

However, not all strategies employed by students are equally effective. Some students adopt a trial-and-error approach, performing transformations without a clear sense of direction or justification. This approach leads to inconsistencies, especially when students experiment with operations that are not structurally justified. As a result, errors such as incorrect distribution, inconsistent handling of negative signs, or unwarranted assumptions about equivalence frequently appear. These errors highlight the need for instructional emphasis on strategic planning rather than mechanical execution.

A notable pattern is that students who struggle often overlook the relational meaning of symbols and focus instead on superficial features of the expression. For example, encountering parentheses often prompts immediate distribution, even in cases where factoring or substitution would be a more appropriate strategy. This behavior indicates a default procedural habit, suggesting that students may not yet view algebraic expressions as objects that can be manipulated flexibly according to their structural properties.

On the other hand, students who engage in relational reasoning demonstrate an ability to view expressions holistically. They consider equivalence not only in terms of visual similarity but as a logical relationship grounded in algebraic principles. These students recognize that transformation is not merely about rewriting expressions but about preserving inherent structure while altering form. Their strategies include substitution of subexpressions, reorganization of terms to reveal latent patterns, and intentional selection of transformation pathways that reduce complexity.

The data also reveal a subset of students who effectively combine procedural fluency with conceptual insight. These students switch flexibly between symbolic manipulation, structural recognition, and strategic simplification depending on the task demands. Their versatility allows them to choose methods that reduce cognitive load while maintaining accuracy, such as converting complicated radicals to exponent forms before expansion or using factoring to reverse incorrect expansion pathways.

To illustrate the variation in strategy use, the following table summarizes the dominant strategies identified in relation to performance indicators observed during the study.

Table 3. Dominant Strategies in Algebraic Expansion and Transformation

Strategy Type	Description	Typical Indicators	Associated Performance Level
Rule-based manipulation	Applying distributive, associative, and commutative rules directly	Correct but rigid transformations; errors when structure is unfamiliar	Moderate
Structure-driven analysis	Identifying underlying patterns before manipulation	Efficient transformations; recognition of algebraic identities	High
Recursive decomposition	Breaking expressions into smaller subexpressions	Stepwise organization; fewer symbolic errors	High
Trial-and-error manipulation	Unsystematic symbolic operations	Inconsistencies; unjustified transformations	Low
Relational reasoning	Viewing expressions holistically and structurally	Flexible strategy switching; accurate simplification	High

Overall, the variety of strategies used by students showcases not only their procedural abilities but also the depth of their conceptual reasoning. Effective strategies tend to reflect a strong awareness of algebraic structures, whereas ineffective ones often arise from procedural dependence and lack of symbolic control. These findings underscore the importance of promoting strategic flexibility in instruction, enabling students to see algebraic expressions not merely as symbols to be manipulated but as structured entities that invite thoughtful and purposeful transformation.

Student Approaches to Nonroutine Problem-Solving

Students' approaches to nonroutine problem-solving reveal a broad range of cognitive strategies that reflect their readiness to engage with unfamiliar mathematical tasks. Unlike routine exercises that rely on memorized procedures, nonroutine problems require students to interpret the structure of the task, explore potential pathways, and make strategic decisions. The data show that students who performed well on these tasks typically exhibited a willingness to analyze the problem deeply before attempting symbolic manipulation, demonstrating cognitive flexibility that aligns with higher-order thinking.

A prominent approach among successful students is exploratory reasoning, where they begin by examining the problem from multiple angles to identify underlying patterns and relationships. These students frequently sketch diagrams, reframe the problem in their own words, or test small cases to gain insight into its structure. This exploratory phase allows them to form preliminary conjectures that guide their next steps, resulting in solutions that are both coherent and justified.

Another effective approach involves strategic decomposition, where students break down a complex nonroutine task into manageable subproblems. By isolating components of the task, they reduce cognitive load and focus on solving smaller pieces that eventually contribute to the overall solution. Students who apply this strategy tend to produce solution pathways that are logically sequenced, demonstrating clear connections between substeps. This method reflects the type of metacognitive planning essential for complex problem-solving.

Some students employ analogical reasoning, attempting to map features of the unfamiliar problem onto problems they have previously encountered. When executed successfully, this approach enables students to repurpose known strategies in novel contexts, producing efficient solutions. However, this approach is only beneficial when students correctly identify relevant similarities; misapplied analogies often lead to incomplete or incorrect solutions, illustrating the fine balance required in transferring prior knowledge.

Students who struggle with nonroutine tasks often rely on trial-and-error techniques, characterized by unsystematic attempts to manipulate expressions or test arbitrary values. This approach lacks strategic grounding and typically produces fragmented, incoherent solutions. The absence of structural understanding becomes apparent in these cases, as students fail to identify which operations are meaningful and which are merely mechanical.

A notable challenge for some students is cognitive rigidity, or the tendency to cling to familiar procedures even when these procedures are inappropriate for the task. These students may attempt to apply routine algebraic techniques, such as standard expansions or substitutions, even when the problem demands pattern recognition, reasoning with generality, or non-algorithmic thinking. This rigidity prevents them from adapting their approach to the unique demands of the problem.

Conversely, students showing higher-order reasoning demonstrate strong metacognitive awareness. They monitor their solution processes, reflect on errors, and adjust their strategies when encountering obstacles. This reflective regulation allows them to pivot between strategies, abandon ineffective paths, and refine their reasoning based on emerging insights. Their solutions are not only correct but also demonstrate a coherent, reflective problem-solving narrative.

The influence of strategy variation on solution quality is evident in the results. Students who employed analytical and flexible strategies tended to produce complete solutions, articulate justifications, and maintain symbolic accuracy. Those who relied on procedural habits or unstructured exploration often produced partial or incorrect answers, marked by gaps in logical reasoning or misuse of algebraic symbols. The relationship between strategy choice and performance highlights the critical role of adaptivity in mathematical problem-solving.

To illustrate the diversity of approaches, the following table summarizes the main patterns of student strategies observed in nonroutine problem-solving tasks.

Table 4. Patterns of Student Approaches to Nonroutine Problem-Solving

Approach Type	Key Characteristics	Strengths	Common Weaknesses

Exploratory reasoning	Examines multiple representations, tests small cases	Deep insight; flexible reasoning	Time-consuming; may lack direction early on
Strategic decomposition	Breaks tasks into subproblems	Organized, structured solutions	Risk of focusing too narrowly on parts
Analogical reasoning	Relates problem to known patterns	Efficient transfer of knowledge	Misapplied analogies lead to errors
Trial-and-error	Unsystematic attempts to manipulate symbols	Occasional correct guesses	Poor structure; inconsistent reasoning
Metacognitive regulation	Monitors and adjusts strategies	High accuracy; coherent reasoning	Requires strong prior knowledge

Quality and Coherence of Mathematical Explanation

The quality and coherence of students' mathematical explanations provide critical insights into their algebraic reasoning processes, particularly when solving tasks involving expression expansion, transformation, and nonroutine problem-solving. The data indicate significant variation in the clarity, structure, and logical justification presented by students. High-performing students generally offer explanations that demonstrate not only procedural correctness but also a deep understanding of the underlying mathematical principles guiding each step. Their reasoning reveals a capacity to articulate connections between symbolic manipulations and conceptual frameworks, forming explanations that are both logically sound and pedagogically meaningful.

One prominent characteristic of coherent explanations is the presence of clear step-by-step articulation. Students who excel in this aspect are able to present their reasoning in a logically ordered sequence, ensuring that each transformation or operation is justified explicitly. They explain why a particular distributive property, factoring rule, or symbolic manipulation is applied, highlighting their awareness of algebraic structure. This form of articulation reflects strong metacognitive skills and demonstrates the ability to communicate mathematical ideas effectively.

Another important feature of high-quality explanations is the use of accurate mathematical language. These students consistently employ appropriate terminology—such as "coefficient," "variable," "equivalent expression," "structural pattern," or "generalization"—to describe their reasoning. Their precise use of language enhances clarity and shows mastery of disciplinary discourse. Students who use mathematical language correctly tend to produce explanations that align well with theoretical expectations of algebraic reasoning.

In contrast, students with weaker explanations often rely on vague or informal descriptions that obscure the logic behind their steps. Their explanations may include statements such as "I just moved it," "I changed it," or "I know the formula," which do not reveal the conceptual basis of their decisions. This lack of specificity suggests a reliance on memorized procedures rather than structural understanding. As a result, their explanations fail to communicate the reasoning needed to justify their solutions, reducing the overall coherence.

Logical justification also plays a central role in assessing explanation quality. Students with strong reasoning support each transformation with valid mathematical arguments, referencing properties such as commutativity, associativity, or distributivity when appropriate. They demonstrate an understanding that algebraic manipulation is not arbitrary but governed by rules that preserve equivalence. Conversely, errors in justification often stem from misconceptions, such as misinterpreting variable roles or overgeneralizing rules. These mistakes reveal underlying cognitive gaps that hinder the development of deeper reasoning.

The ability to use representations appropriately further distinguishes high- from low-quality explanations. Students with strong conceptual understanding incorporate tables, symbolic breakdowns, or diagrams to support their reasoning when necessary. These representations serve as tools for clarifying or reinforcing their arguments. Meanwhile, students who struggle tend to avoid auxiliary representations altogether or use them incorrectly, leading to fragmented and sometimes contradictory explanations.

Coherence is also influenced by students' ability to connect intermediate steps to the overall goal of the problem. Strong students consistently relate their manipulations back to the objective—such as simplifying an expression, identifying structure, or solving an unknown. This global awareness ensures that their explanations form a unified narrative rather than a collection of disconnected steps. Weak explanations, by contrast, often display a lack of direction, with students performing symbolic operations without articulating how those steps contribute to the final solution.

The data also reveal differences in how students handle errors within their explanations. Students with mature reasoning acknowledge mistakes, revise their steps, and incorporate the correction into their explanation. This reflective behavior strengthens the coherence of their reasoning. Students with weaker reasoning either fail to recognize inconsistencies or leave contradictions unresolved, producing explanations that lack internal logic.

To synthesize the findings, the table below summarizes the primary dimensions that distinguish high-quality from low-quality mathematical explanations among participants.

Table 5. Key Dimensions of Quality and Coherence in Students' Mathematical Explanations

Dimension	Characteristics of High-Quality Explanations	Characteristics of Low-Quality Explanations
Logical structure	Clear sequence; each step justified	Disorganized steps; missing justification
Mathematical language	Precise terminology; correct usage	Vague wording; incorrect or informal terms
Conceptual justification	Reasoning tied to algebraic properties	Reliance on memorized procedures
Representation use	Appropriate, supportive diagrams or tables	Little or no representation; incorrect use
Global coherence	Steps aligned with overall goal	Disconnected steps; unclear purpose

Overall, the quality and coherence of students' mathematical explanations provide a meaningful indicator of their algebraic reasoning abilities. Students who combine structural awareness, precise language, and clear justification demonstrate reasoning aligned with established theories of advanced algebraic thinking. These findings highlight the importance of instructional practices that emphasize explanation, justification, and mathematical communication, ensuring that students not only perform procedures but also understand and articulate the reasoning that legitimizes those procedures.

6. Conclusions

The conclusions of this study highlight the complexity and diversity of students' algebraic reasoning processes when engaging with tasks involving expression expansion, symbolic transformation, and nonroutine problem-solving. The findings demonstrate that students with strong conceptual understanding exhibit flexible, strategic, and reflective reasoning that enables them to navigate unfamiliar mathematical challenges effectively. Their problem-solving approaches are characterized by coherent explanations, precise mathematical language, and the ability to justify each transformation based on underlying algebraic principles. This group consistently shows the capacity to generalize patterns, interpret symbolic relationships, and maintain structural awareness—key indicators of advanced algebraic reasoning.

In contrast, students who rely heavily on procedural memorization encounter persistent difficulties when required to apply reasoning beyond routine tasks. Their explanations often lack clarity, coherence, and justification, indicating gaps in conceptual understanding. Misconceptions and errors commonly arise from overgeneralized rules, limited structural recognition, and inconsistent symbolic manipulation. These patterns highlight the importance of fostering deep conceptual understanding rather than focusing solely on procedural fluency in mathematics instruction at the university level.

Overall, the study underscores the need for instructional approaches that prioritize reasoning, explanation, and structural understanding in algebra. Effective teaching practices should encourage students to explore multiple solution strategies, articulate their reasoning explicitly, and engage with mathematical tasks that promote generalization and flexible thinking. By integrating these elements into learning environments, educators can better support students in developing the level of algebraic reasoning necessary for advanced mathematical study and for solving complex, nonroutine problems.

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